Total No. of Questions : 9]

SEAT No. :

PA-1182

[Total No. of Pages : 7

[5925]-204
S.E. (Civil)
ENGINEERING MATHEMATICS - III
(2019 Pattern) (Semester - III) (207001)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Question No. 1 is compulsory.***
2) ***Attempt Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.***
3) ***Assume suitable data, if necessary.***
4) ***Neat diagrams must be drawn wherever necessary.***
5) ***Figures to the right indicates full marks.***
6) ***Use of electronic pocket calculator is allowed.***

Q1) a) The pair of regression Linens are L1 : $8x - 10y + 66 = 0$ and L2 : $40x - 18y = 214$ **[1]**

i) L1 is the regression Line y on x.

ii) L1 is the regression line x on y.

iii) L2 is regression line y or x.

iv) L1 and L2 is regression line x on y.

b) Vector along the direction of the line. **[1]**

$\frac{x-1}{2} = \frac{y+2}{1} = \frac{z-3}{5}$ is

i) $\frac{\hat{i} - 2\hat{j} - 3\hat{k}}{\sqrt{14}}$

ii) $\frac{\hat{i} + 2\hat{j} + 5\hat{k}}{\sqrt{30}}$

iii) $\frac{2\hat{i} + \hat{j} - 5\hat{k}}{\sqrt{30}}$

iv) $\frac{2\hat{i} + \hat{j} + 5\hat{k}}{\sqrt{30}}$

c) Let $X = B(7, 1/3)$ be the Binomial distribution with parameters n = 7 and p = 1/3. Then $p(x=2)+p(x=5)$ is **[2]**

i) 81/28 ii) 28/81

iii) 7/81 iv) 10/81

d) If vector field $\vec{F} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+mz)\hat{k}$ is solenoidal the value of m is **[2]**

i) -2 ii) 3

iii) 2 iv) 0

e) Using Stoke's theorem $\oint_c \vec{F}\cdot d\vec{r}$ where $\vec{F} = xy\hat{i} + yz\hat{j} + z\hat{k}$ over the cube whose side is a and it's face in XOY - plane is missing is equal to **[2]**

i) 0 ii) $\iint_R y\,dxdy$

iii) $\iint_R 2x\,dxdy$ iv) $\iint_R -x\,dxdy$

f) Most general solution of $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ is **[2]**

i) $u(x,t) = (c_1 \cos mx + c_2 \sin mx)(c_3 \cos cmt + c_4 \sin cmt)$

ii) $u(x,t) = (c_4 \cos mx + c_5 \sin mx)e^{-m^2 t}$

iii) $u(x,t) = (c_1 e^{-mx} + c_2 e^{mx})(c_1 \cos my + c_2 \sin my)$

iv) $u(x,t) = (c_1 \cos mx + c_2 \sin mx)(c_3 e^{-my} + c_4 e^{my})$

Q2) a) A computer while calculating carrelation coefficient between two variables X and Y from 25 pairs of observations obtained the following results :

$n = 25, \ \sum X = 125, \ \sum X^2 = 650, \ \sum Y = 100, \ \sum Y^2 = 460, \ \sum XY = 508.$

Later it was discovered that the values (X, Y) = (8, 12) was copied as (6, 14) and the value (8, 6) was copied as (6, 8). Obtain the correct value of the correlation coefficient. **[5]**

b) In a normal distribution 31% of the items are under 45 and 8% are above 64. Find the mean and standard deviation of the distribution. Take Area $(0 < z < 1.4) = 0.42$ and Area $(0 < z < 0.5) = 0.19$ where z is the standard normal variate. **[5]**

c) Verify at 5% level of significance and 4 degrees of freedom if the distribution can be assumed to be poisson given:

# defects :	0	1	2	3	4	5
Frequency :	6	13	13	8	4	3

Take $e^{-2} = 0.135.$ in the calculations round off the frequencies to the immediate higher integral value. Take $\chi^2_{5,0.05} = 11.07$ **[5]**

OR

Q3) a) Two examiners A and B award marks to seven students as follows:

Roll No. :	R_1	R_2	R_3	R_4	R_5	R_6	R_7
Marks (A) :	40	44	28	30	44	36	30
Marks (B) :	32	39	26	30	28	34	28

Find the coefficient of correlation. **[5]**

b) Assume the mean height of soldiers to be 68.22 inches with a variance of 10.8 inches square. How many soldiers in a regiment of 10,000 would you expect to be over 6 feet? Assume area $(0 < z < 1.15) = 0.3749$ where z is the standard normal variate. **[5]**

c) Among 64 off springs of a certain cross between European horses 34 were red, 10 were black and 20 were white. According to a genetic model these numbers should be in the ratio 9:3:4. Is the data consistent with the model at 5% level of significance? Take $\chi^2_{2;0.05} = 5.991$ **[5]**

Q4) a) Find the angle between the tangents to the curve **[5]**

$x=t, y=t^2, z=t^3 \neq$ at $t=1$ and $t=-1$

b) If $\vec{F}_1 = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$ and $\vec{F}_2 = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ then show that $\vec{F}_1 \times \vec{F}_2$ is solenoidal. **[5]**

c) If the directional derivative of $\phi = axy + byz + czx$ at (1, 1, 1) has maximum magnitude 4 in a direction of *x-axis*. Find a, b and c. **[5]**

OR

Q5) a) Find the directional derivative of $\phi = xy + yz^2$ at the point (1, –1, 1) to wards point (2, 1, 2). **[5]**

b) Prove the following identities (any one) **[5]**

i) $\nabla \times (\bar{a} \times \bar{r}) = 2\bar{a}$

ii) $\nabla(\vec{a}.\vec{r}) = \vec{a}$

c) Show that $\vec{F} = (xy^2 + xz^2)\hat{i} + (yx^2 + yz^2)\hat{j} + (zx^2 + zy^2)\hat{k}$ is irrotational. Find scalar ϕ such that $\vec{F} = \nabla\phi$. **[5]**

Q6) a) Evaluate $\int_c \bar{F} \cdot d\bar{r}$ along the straight line joining points (0, 0, 0) and (2, 1, 3) where $= \bar{F} = 3x^2\bar{i} + (2xz - y)\bar{j} + z\bar{k}$ **[5]**

b) Evaluate $\iint_S (x\bar{i} + y\bar{j} + z\bar{k}) \cdot d\bar{s}$ over the surface of sphere $x^2 + y^2 + z^2 = 1$ **[5]**

c) Evaluate using Stoke's theorem $\iint_S (\nabla \times \bar{F}) \cdot d\bar{s}$ where $\bar{F} = y^2\bar{i} + z\bar{j} + xy\bar{k}$ and S is surface of paraboloid $z = 4 - x^2 - y^2 (z \geq 0)$. **[5]**

OR

Q7) a) Use Green's theorem to evaluate $\int_c (2x^2 - y^2)dx + (x^2 + y^2)dy$ where 'C' is boundary of area enclosed by the axis and circle $x^2 + y^2 = 16, z = 0.$ **[5]**

b) Apply Stoke's theorem to evaluate $\int_c \overline{F} \cdot d\overline{r}$ where $\overline{F} = yz\overline{i} + zx\overline{j} + xy\overline{k}$ and S is upper part of sphere $x^2 + y^2 + z^2 = 1$ above XOY plane. **[5]**

c) Evaluate $\iint_s (x\overline{i} + y\overline{j} + z^2\overline{k}) \cdot d\overline{s}$. Where S is the surface of cylinder $x^2 + y^2 = 4$ bounded by planes $z = 0$ and $z = 2$. **[5]**

Q8) a) A string stretched and fastened between two points L a part. Motion is started by displacing the string in the form $y = a \sin \frac{\pi x}{L}$ from which it is released at time $t = 0$. Find the displacement $y(x,t)$. **[8]**

b) Solve the one dimensional heat equation $\frac{\partial y}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ subject to conditions.

i) u is finite $\forall t$.

ii) u (0, t) = 0,

iii) u(π , t) = 0,

iv) u(x, 0) = $\pi x - x^2 \quad 0 \le x \le \pi.$ **[7]**

OR

Q9) a) A tightly stretched string with fixed ends $x = 0$ and $x = l$ is initially at rest in its equilibrium position is set to vibration by giving each point a velocity $3x(l - x)$ for $0 < x < l$. Find the displacement $y(x, t)$ at any time t. **[8]**

b) An infinitely long uniform metal plate is enclosed between lines $y = 0$, and $y = l$ for $x > 0$. The temperature is zero along the edges $y = 0$, $y = l$, and at infinity. If edge $x = 0$ is kept at a constant temperature v_0, Find the temperature distribution $v(x, y)$. **[7]**